Benchmark for advection solver based on particles method

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Some benchmark are provided in order to measure the implementation efficiency, the drawback and the benefits of our numerical method and there precision. These tests are design in order to present "easy" cases as complex cases hard to simulate.

1 Mathematical description of the benchmarks

Different benchmark are provided. The purpose is to evaluate the advection solver and its implement in both simple cases and complex cases.For reader convenience, let us sum up all these benchmark :

1. 2D turning sphere : simple case where analytic solution are known.
2. Radial constant field with a velocity involving shear: this test-case allows to test how efficient the solver is in complex cases with large time step. The analytic solution is known.

1.1 (2D-)Turning sphere

Let us denote by $\Omega = [0,1]^2$ the numerical domain.

The velocity field is defined by:

$$\mathbf{v}\left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) = \frac{2\pi}{T} \begin{pmatrix} 0.5 - y \\ x - 0.5 \end{pmatrix} \tag{1.1}$$

with the period $T = 1$ and $r_0 = \min(dx, dy)$ where dx, dy denote the space step . The scalar is initialized as following :

$$u\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \begin{cases} 0 & \text{if } r^2 = \left(x - 3/5\right)^2 + \left(y - 3/5\right)^2 > r_0^2 \\ \left(1 - r^2/r_0^2)^4\right) & \text{else, with } r^2 = \left(x - 3/5\right)^2 + \left(y - 3/5\right)^2 \end{cases} \tag{1.2}$$

The analytic solution at time t consists of a rotation of the initial scalar value along Z-axis of angle $\frac{2\pi}{T} \times t$.

1.2 Radial constant field with a velocity involving shear

Let us denote by $\Omega = [-1, 1]^2$ the numerical domain.
The velocity field is defined by:

$$\mathbf{v}\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \cos\left(\frac{3\pi}{2}\right)\left(\begin{smallmatrix} -y \\ x \end{smallmatrix}\right) \tag{1.3}$$

The scalar is initialized as following :

$$u\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = \begin{cases} 0 & \text{if } r^2 = x^2 + y^2 > 1 \\ \left(1 - r^2)^6\right) & \text{else, with } r^2 = x^2 + y^2 \end{cases} \tag{1.4}$$

As the radial component of the velocity vanishes, the initial scalar value match to a stationary solution. Therefore, the scalar field remains constant. As there is some tangent shear, this test will show how the implementation deals with large time step. For instance, if it is based on $\Lambda_{\tilde{2}}$ or $\Lambda_{\tilde{4}}$ remeshing formula, the corrected cases will appears (*ie* some particles will be tagged). Note that for a CFL number larger than one, classical λ_6 formula are only of order 1.